SOLUTION OF PROBLEM 206.

BY R. J. ADCOCK, MONMOUTH, ILLINOIS.

EDITOR ANALYST:

Having given at pages 183–4, Vol. IV, ANALYST, a correct demonstration of the principle of Least Squares, and at pages 21–22, Vol. V, the derivation of formulæ for mean error, probable error, &c., and on pages 53–54, the solution of an example according to this corrected method, and not having made a single disciple, as appears from the published solution at p. 122, of my second ex., problem 206: I therefore propose to send you a correct solution of that example, 206, for publication in No. 5, if you will insert it, and will state my objection to Prof. S's solution.

R. J. ADCOCK.

[As Mr. Adcock has nowhere, so far as we know, explicitly pointed out the *error* which his treatment of the subject is intended to correct, we have given, above, his communication in full, and subjoin the solution and objections referred to.—Ed.]

Solution.—Let $x_1, y_1, z_1, x_2, y_2, z_2, \ldots x_n, y_n, z_n,$ be the coordinates of the n points, and

$$z = cx+dy+g, \tag{1}$$

the equation of the required plane, in which the quantities c, d and g are to be determined when the plane is in its most probable position.

Now it is demonstrated at page 183–4 ANALYST, Vol. IV, that any point, line or surface, to be determined from the measured coordinates of n points, has its most probable position when the sum of the squares of the normals upon it from the n points is a minimum.

By Analytical Geometry, the normal from the point $x_1, y_1, z_1,$ to the plane whose equation is (1) is

$$N_1 = \frac{cx_1+dy_1-z_1+g}{(1+c^2+d^2)^{\frac{1}{2}}}. \tag{2}$$

Hence, writing $S(N_1^2) = N_1^2+N_2^2+\ldots+N_n^2$, &c., and $S(x_1\,y_1)$
$= x_1y_1+x_2y_2+\ldots x_ny_n$, &c.,

$$S(N_1^2) = \frac{c^2S(x_1^2)+d^2S(y_1^2)+S(z_1^2)+ng^2+2cdS(x_1y_1)}{1+c^2+d^2}$$
$$+\frac{2cgS(x_1)-2dS(y_1z_1)+2dgS(y_1)-2cs(x_1z_1)-2gS(z_1)}{1+c^2+d^2} \tag{3}$$

Then, from (3), c, d and g are to have such values as make $S(N_1^2)$ a minimum. Therefore

$$D_cS(N_1^2) = 0\,..\,(4), \quad D_dS(N_1^2) = 0\,..\,(5), \quad D_gS(N_1^2) = 0, \tag{6}$$

are the equations which determine c, d and g; the solution of which can be performed by equations of the second degree, by first transferring the origin of coordinates to the centre of gravity of the n points.

My objection to Prof. Scheffer's solution is, that it does not recognize the proposition in relation to normals above referred to, nor the necessary and fundamental definition of the error of a point with respect to a line or surface, which definition is, that the error is the normal from the point to the line or surface.

SOLUTIONS OF PROBLEMS IN NUMBER FOUR.

SOLUTIONS of problems in No. 4 have been received as follows:

From Prof. W. W. Beman, 215; Marcus Baker, 213, 215, 217; George C. Comstock, 215; Geo. M. Day, 212, 213, 214, 215, 216, 217, 219; Prof. A. B. Evans, 213; Geo. Eastwood, 218; Henry Gunder, 212, 213, 214, 217; W. E. Heal, 212, 213, 214; Wm. Hoover, 212, 213, 214; Henry Heaton, 212, 213, 214, 215, 216, 217, 219, 220; George H. Harvill, 213; Prof W. W. Johson, 217, 220; Prof. J. H. Kershner, 212, 213, 214, 215, 216, 217, 219; Prof. D. J. Mc Adam, 212, 213, 214, 217; L. W. Meech, 218; Artemas Martin, 220; Prof. J. Scheffer, 212, 213, 214, 215, 216, 217; E. B. Seitz, 212, 213, 214, 215, 216, 217, 219, 220.

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211. " Prove that every number is either a triangular number or is the sum of two, or of three triangular numbers."

[No demonstration of this proposition has been received. Prof. Scheffer writes: "This theorem is a special case of a more general one. As to this case and the following one, viz.: *Every number is either a square or the sum of two, three, or four squares,* the demonstrations have been discovered, but as to the pentagonal and higher numbers, the attempts of the greatest mathematicians at a demonstration have thus far proved futile."

The writer of the article, *number*, in *Johnson's New Encyclopædia,* says, "It is a general principle, though not capable of rigorous demonstration, that any whole number is equal to the sum of 1, 2 or 3 triangular numbers, or to the sum of 1, 2, 3 or 4 square numbers, or to the sum of 1, 2, 3, 4, or 5 pentagonal numbers, etc."]

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212. "One half of a circular tract of land is cut off by an arc of a circle whose center is in the circumference of the circular tract. Find the radius with which the arc is described."